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Series-IC-Enter-A The Shredder Interface and Conditional Horizon Access

Consolidation: sharp floors, imperfect access, parent comparison, compact conjectures, and endpoint non-selection

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Abstract

We consolidate a self-contained finite-channel synthesis of the shredder interface and its conditional comparison with observer access on a supplied classical cosmological background. Every local theorem is retained: classical cut $1 - 1/d$, residual dimension $1 - k/d$, replacement $1 - 1/d^2$, the flagged Weyl identity, imperfect-flag optima, spectral approximate reset, private-key entropy, the heralded-cut obstruction, thermal underdetermination, and the rank obstruction to exact pure reset.

Repeated cosmic and horizon comparison proofs are shortened to local statements with named sibling comparisons; the co-located epoch counterexample stays in full. A parent-comparison table records source labels as records, not premises; imperfect-access proofs are tightened; a key-reset heat remark links private-key entropy to thermal underdetermination without a universal heat-per-qubit law. Eight research conjectures are compacted to a completion-criterion table with short scopes; none is promoted.

No spacetime emergence, Einstein dynamics, Page curve, island, bounce, vacuum-energy selection, or Host C is asserted. O.1 stays NOT_CLOSED; Gate 0/ST4b NOT_SUPPLIED; Dom (T) undetermined; J2 unresolved.

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Routing, philosophy, and scope

Identity, statement types, and organizing separation

The identifier is Series-IC-Enter-A, with alias IC-Enter-A. This file is a documentary synthesis with locally proved mathematical results, not a registration request:

$$\text{declaredPath} = \text{null}, \qquad \text{wired} = \text{false}.$$

Standing boundary. No spacetime emergence, derived metric, Einstein dynamics, Page curve, island, cosmological bounce, de Sitter quantum state, vacuum-energy selection, inherited characteristic quantum coefficient, compilation, or Host C is claimed. O.1 stays NOT_CLOSED; Gate 0 and ST4b stay NOT_SUPPLIED; XV-B is not opened; XVI-N is not discharged; Dom (T) remains undetermined; J2 remains unresolved; Problems 6, 7, and 9 are named, not solved. Locked Host-A stays NAMED_COMPARISON_ONLY, with $\{\mathrm{GI}, \mathrm{CB}\}$ only. Cited Cheung and OpenAI records are source records, not premises. The declared path is null; wired=false.

We distinguish three roles:

1. *Supplied data*: a finite code, complete instrument, accessible output algebra, metric, calibration, reservoir model, or allowed continuation class, printed explicitly here.

2. *Locally proved consequence*: an implication proved below from those displayed data and hypotheses.

3. *Named comparison*: a source record giving context and provenance, without transferring its theorem as an unstated premise.

A theorem below is a mathematical implication. A physical prediction would additionally require a same-model preparation, interaction, acquisition, calibration, localization, and complete implementation-error contract. A research conjecture concerns such an unsupplied bridge and is not used in any proof.

The central separation is

$$\mathcal{M}(\rho) = \sum_r \mathrm{proj}\, r_R \otimes \mathcal{N}_r(\rho) \quad \neq \quad \mathcal{C}(\rho) = \mathrm{Tr}_A \mathcal{M}(\rho)$$

as access contracts when A remains available. Writing a classical record does not, by itself, discard every quantum output. Likewise, observing a thermal-looking marginal does not prove that the complete accessible output is classical.

The lengthy characteristic-product excerpts supplied with the plan concern a different analytic target. No characteristic map, extension prescription, or quantum Ward identity is constructed by the finite recovery proofs below. Their unresolved admission boundary is retained, not addressed by renaming a report channel.

Enter, spacetime Enter, and the Shredder Philosophy

In the Enter interpretation, a live quantum name is an owned quantum resource. Destructive readout consumes that name in the continuation's quantum context, returns a classical word, and may leave unmeasured quantum systems available [4, 2, 5]. Consumption is an output-type and access statement. It is not annihilation of every microscopic apparatus degree of freedom.

Spacetime Enter adds locations, routes, port algebras, clocks, and resource ledgers to a *supplied* host [5, 16]. Its classical spacetime is not a quantum factor consumed by readout. A history code and the coordinate of the reading event are different objects; deleting a clock encoding does not delete the geometric time axis.

The Shredder Philosophy asks which complete outputs remain accessible after a specified cut [6, 12]. A report of an original, a retained quantum residual, and an input-independent replacement are not interchangeable. Global preservation by an isometry does not imply recovery from a restricted classical exterior. Conversely, failure of recovery from that exterior does not prove global destruction.

The relevant distinctions are:

report law ≠ complete instrument,
accessible loss ≠ global loss,
readout ≠ reset ≠ reservoir heat,
finite code dimension ≠ horizon entropy,
prefix agreement ≠ terminal ownership.

The Big-series analogy

The fixed-size or "box" analogy compares a declared finite resource with an observer-dependent causal region on a prescribed classical host [19, 20, 21, 22, 23]. It becomes mathematical only after the compared objects and their interfaces are specified. Finite geometric size does not imply a finite number of field modes or a finite-dimensional quantum state space.

For this synthesis, Big01 supplies the named prescribed-horizon access comparison; Big02–Big05 supply documentary comparisons separating capacity accounts, source accounts, coefficient-sensitive labels, radiative accounts, and constitutive interpretation. No theorem here identifies the mathematical shredder with a Big Bang, a black hole, or a cosmological state.

An initial allocation is not a cosmological singularity. An internal reorganization is not a derived expansion law. A resource reset is not a cosmological bounce. The analogy motivates an audit of missing inputs, not a physical identification.

toprule Source Named comparison Boundary retained

midrule endhead I-C Classical floor, append-only entropy, thermal ledger, complete exterior, prefix endpoints Access is not a Page curve or gravitational endpoint [6].

I-C-Ex / I-C-Ex2 Residual dimension, complete complement, measurement-based computation Returned quantum access differs from a classical marginal [7, 8].

I-C-CERT Replacement spectral law and access certificates $1 - 1/d^2$ is not $1 - 1/d$ [9].

I-C-Th Reset, thermal, and underdetermination accounts Heat does not select a source or vacuum [10, 11].

I-B / I-B-G / I-D Complete Enter instrument and host distinctions Metric and operator realizations are additional data [5, 14, 15, 16].

En09-B Finite-clock access and sharp robustness ratio Its state-ratio constant is not a new geometric coefficient [17, 18].

Big01-A Horizon replacement, labels, vacuum fibres, future assumptions Prescribed classical host only [19].

Big-Vac-A / Big06-B Vacuum non-selection and source residual No vacuum or particle is identified [25, 26, 27, 24].

Earlier synthesis sheets Finite-cut and cosmic-reach assembly style Style and provenance only [28, 29, 30].

Standing records State, source-domain, finite-Hilbert, and analytic gates No missing gate is supplied [35, 36, 37, 38, 39, 31, 32, 33, 34].

OpenAI catalogue Documentary external-catalogue comparison No expansion law, shredder–horizon identification, or coefficient is imported [50].

bottomrule

Parent-comparison table (local theorems to named sources)

Not a premise / style-only notes for I-C-Ex, I-C-Ex2, I-C-Th, I-C-Th-CERT, Big-Vac-A, and Series syntheses are recorded in this parent-comparison table, not inside bibliography entries (book pass t415u).

The following table maps retained local theorems to named parent records. Every cited Cheung or OpenAI label is a *source record*; local hypotheses are printed independently and are not transferred as unstated premises.

toprule Local theorem Named parent / extension Retained boundary

midrule endhead thm:A I-C thm:A Access, not horizon entropy

thm:sharp I-C-Ex thm:sharp Residual dimension is not a logical code

thm:replacement I-C-CERT thm:replacement $1 - 1/d^2$ is not $1 - 1/d$

thm:imperfect-flag extends local thm:flagged Not a horizon model

thm:approximate-reset-spectrum extends I-C-Th thm:reset Unrestricted unitary bound only

thm:heralded-cut *not* I-C-Ex2 thm:mbqc Exact heralded return ≠ MBQC

thm:cDsharp local correction of En09-B state-ratio Not a channel floor; bandwidth ≠ CCR

bottomrule

The prescribed reporting carrier

The I-C reporting comparison uses the regular conical product

$$\begin{aligned} M_{\alpha} &= \mathbb{R}_{T} \times (0,\infty)_{r} \times (\mathbb{R}/2\pi\mathbb{Z})_{\varphi} \times \mathbb{R}_{z}, \\ g_{\alpha} &= -c^{2}\mathrm{mathop\,d}T^{2} + \mathrm{mathop\,d}r^{2} + \alpha^{2}r^{2}\mathrm{mathop\,d}\varphi^{2} + \mathrm{mathop\,d}z^{2}, \qquad 0 < \alpha < 1, \quad c > 0. \end{aligned}$$

Its declared completion adds the worldsheet $\mathscr{W} = \mathbb{R}_{T} \times \{o\} \times \mathbb{R}_{z}$. No smooth metric on that missing locus is assigned.

Theorem (Prescribed cone and complete destructive reporting). *The regular metric is locally flat with signature* $(-+++)$*, has angular deficit* $2\pi(1-\alpha)$*, and is not globally hyperbolic.*

For an orthonormal clock basis $\{|\tau_{j}\rangle\}_{j=0}^{D-1}$ *and a finite survivor A, put*

$$D_{j} = \langle \tau_{j}|_{C} \otimes I_{A}, \qquad \mathscr{M}_{C}(X) = \sum_{j} \mathrm{proj}\, j_{R} \otimes D_{j} X D_{j}^{\dagger}.$$

Then $\mathscr{M}_C$ is CPTP and, with every untouched reference E,

$$\mathrm{Tr}_R\,[(\mathscr{M}_C \otimes \mathrm{id}_E)(X)] = \mathrm{Tr}_C X.$$

The quantum output contains A, not the consumed clock C. The host and T remain prescribed data.

Proof. The local development

$$(T, r, \varphi, z)\mathrm{longmapsto}\,(cT, r\cos(\alpha\varphi), r\sin(\alpha\varphi), z)$$

pulls back the Minkowski metric to . The circumference at radius r is $2\pi\alpha r$.

Choose $r_* > 0$, fixed φ_*, z_*, and $b > r_*/c$. The diamond between

$$p = (-b, r_*, \varphi_*, z_*), \qquad q = (b, r_*, \varphi_*, z_*)$$

contains $(0, r_*/n, \varphi_*, z_*)$. Radial travel has length less than cb. The sequence has no convergent subsequence in the regular manifold, so the diamond is noncompact.

The recorded Kraus maps are $|j\rangle_R \otimes D_j$; their squared moduli sum to I_{CA}. In $X = \sum_{j,k} |\tau_j\rangle\langle\tau_k| \otimes X_{jk}$, the output blocks are X_{jj}, whose sum is $\mathrm{Tr}_C X$. The same computation includes E. Geometric data do not occur as output quantum factors.

par

The reporting cone is neither full I-D dynamical host. Deleting an axis or keeping only a settled exterior changes the model. Green operators across the missing worldsheet require separately specified domains or realizations [14, 15, 48, 49]. Non-global hyperbolicity is not a theorem that every equation lacks a unique inverse.

Only positive-probability blocks are normalized. Such normalization describes a selected state, not a deterministic channel on arbitrary inputs. All failures, stopping outcomes, and actual classical controls must remain in the complete finite history.

The shredder cut: three sharp access contracts

Finite task and complete interface

Fix an unknown code $Q \simeq \mathbb{C}^d$, $d \geq 2$, and an intended output identification $Q' \simeq Q$. A decoder is deterministic and CPTP. It has no access to the testing reference and no unlisted quantum assistance.

For channels on common input and output spaces define

$$d_\diamond(\Phi, \Psi) = \frac{1}{2}\|\Phi - \Psi\|_\diamond.$$

This includes arbitrary untouched references. The normalized Bell vector and its projector are

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |i\rangle_E |i\rangle_Q, \qquad \Phi_d = |\Phi_d\rangle\langle\Phi_d|.$$

For a decoded channel $\Psi : Q \to Q'$, write

$$F_{\mathrm{e}}(\Psi) = \mathrm{Tr}\,[\Phi_d(\mathrm{id}_E \otimes \Psi)(\Phi_d)].$$

For an accessible channel $\mathcal{N}$, put

$$F_{\mathrm{e}}^{\star}(\mathcal{N}) = \sup_{\mathcal{D}} F_{\mathrm{e}}(\mathcal{D}\mathcal{N}), \qquad \varepsilon^{\star}(\mathcal{N}) = \inf_{\mathcal{D}} d_\diamond(\mathcal{D}\mathcal{N}, \mathrm{id}_Q).$$

The Bell projector is an effect, so

$$d_\diamond(\Psi, \mathrm{id}_Q) \geq 1 - F_{\mathrm{e}}(\Psi).$$

For Kraus operators K_j,

$$F_{\mathrm{e}}(\Psi) = \frac{1}{d^2} \sum_j |\mathrm{Tr}\, K_j|^2.$$

Indeed, $\langle\Phi_d|(I \otimes K_j)|\Phi_d\rangle = \mathrm{Tr}\, K_j / d$.

Definition (Complete access cut). A finite classical–quantum cut is a channel

$$\mathcal{M}(X) = \mathrm{bigoplus}_r \mathcal{N}_r(X), \qquad \mathcal{N}_r : \mathcal{B}(Q) \to \mathcal{B}(K_r),$$

where every $\mathcal{N}_r$ is CP and their trace sum is $\mathrm{Tr}\, X$. The index r is classical. The dimension $k_r = \dim K_r$ counts the *complete quantum block accessible to the decoder*.

A complete classical exterior cut has $k_r = 1$ for all r. Its joint history includes outcome, time, failure, stopping, control, and finite heat-meter labels. No quantum bath, receiver memory, shared entanglement across the cut, or other assisting quantum factor bypasses this factorization. A deterministic ledger computed from the history is only classical postprocessing.

This is a whole-code one-use task. Independently prepared additional copies, a different code, a heralded task, or a retained assisting factor changes the contract.

A fixed input-independent ancilla may be absorbed into a decoder. Pre-shared entanglement enabling teleportation cannot be erased from a complete cut description: the joint source-to-receiver factorization is then not the asserted standalone classical channel.

Finite CP maps admit Kraus representations by spectral decomposition of their positive Choi matrices. Conversely, vectorizing Kraus operators gives a positive Choi matrix. These elementary matrix facts fix the finite framework [40].

Complete classical access

Theorem (Sharp classical-record floor). *For every complete finite classical-output channel $\mathscr{C}$ and every deterministic decoder,*

$$F_{\mathrm{e}}(\mathscr{D}\mathscr{C}) \le \frac{1}{d}, \qquad d_{\diamond}(\mathscr{D}\mathscr{C}, \mathrm{id}_Q) \ge 1 - \frac{1}{d}.$$

Both constants are sharp over this class. For the different Haar-average pure-state task,

$$F_{\mathrm{av}}(\mathscr{D}\mathscr{C}) \le \frac{2}{d+1},$$

also sharply.

Proof. There is a POVM $\{E_r\}$ with $\mathscr{C}(X) = \sum_r \mathrm{Tr}\,(E_r X)\mathrm{proj}\, r$. The decoder prepares a state σ_r on each classical

branch. Therefore

$$F_{\mathrm{e}}(\mathscr{D}\mathscr{C}) = \frac{1}{d^2}\sum_r \mathrm{Tr}\,(E_r\sigma_r) \le \frac{1}{d^2}\sum_r \mathrm{Tr}\,E_r = \frac{1}{d}.$$

Equation gives the distance bound.

Measure and reprepare a fixed basis, producing dephasing

$$\Delta(X) = \sum_i \mathrm{proj}\, i X \mathrm{proj}\, i = \frac{1}{d}\sum_{j=0}^{d-1} \mathrm{Ad}_{Z^j}(X), \quad Z = \sum_i e^{2\pi i\, i/d}\mathrm{proj}\, i.$$

Its Bell fidelity is $1/d$. The triangle inequality gives $d_\diamond(\Delta, \mathrm{id}) \le (d-1)/d$, matching the lower bound.

For a general channel, the Haar second moment is

$$\int \mathrm{proj}\,\psi \otimes \mathrm{proj}\,\psi\, \mathrm{mathop\,d}\psi = \frac{I + \mathrm{S}}{d(d+1)},$$

where S swaps the two factors. Unitary invariance makes the integral proportional to the symmetric projector, and its trace fixes the coefficient. Applying this identity to the Kraus representation gives

$$F_{\mathrm{av}}(\Psi) = \frac{d + \sum_j |\mathrm{Tr}\,K_j|^2}{d(d+1)} = \frac{1 + dF_{\mathrm{e}}(\Psi)}{d+1}.$$

Dephasing attains the resulting bound.

par

Immediate scope. These are access contracts, not horizon entropies. For a qubit the referenced channel floor is 1/2, whereas the Haar-average infidelity floor is 1/3. Known orthogonal states can be measured and re-prepared perfectly. Neither tomography on independent copies nor removal of failure trials changes the theorem into universal deterministic quantum return.

Qubit ledger ($d = 2$; not a new theorem). Specialise Theorems – to a qubit. The three sharp floors collapse to the concrete ledger

$$1 - \frac{1}{d} = \frac{1}{2}, \qquad 1 - \frac{k}{d} = 1 - \frac{k}{2}, \qquad 1 - \frac{1}{d^2} = \frac{3}{4}.$$

Thus a complete classical-record channel on a qubit has diamond error at least 1/2; a residual of dimension k retains the cut law $1 - k/2$; and the complete-access / replacement floor is 3/4. For $k = 1$ the middle entry equals 1/2, matching the classical-record floor; for $k = 0$ it equals 1, matching total erasure. These are the same three constants already proved sharp in the parent theorems; the ledger is a numerical specialisation, not a promotion of any research conjecture, and it does not identify the shredder interface with a Gibbons–Hawking state. The same ledger at $d = 3$ reads $\frac{2}{3}$, $1 - k/3$, $\frac{8}{9}$; at $d = 4$ it reads $\frac{3}{4}$, $1 - k/4$, $\frac{15}{16}$. Recording the $d = 2$ row explicitly is enough for the book expansion, because every later numerical comparison in this sheet specialises to a qubit whenever a concrete device is named. For a flagged Weyl mixture on a qubit with perfect delivery probability $p_{\mathrm{ok}} = 0.9$ and a single replacement branch of weight 0.1, the complete-access floor $\frac{3}{4}$ remains the deterministic lower bound on diamond error in the absence of the flag, while the flagged theorem improves the error to 0.1 when the flag is read. The numerical gap $0.75 - 0.10 = 0.65$ is exactly the value of reading the delivery flag inside the declared class. None of these arithmetic lines promotes a conjecture, identifies the shredder with Gibbons–Hawking thermodynamics, or imports a horizon entropy as an access contract.

This is the local comparison with I-C's thm:A [6], consistent with entanglement-breaking channel theory [41].

Retained residual dimension

Theorem (Sharp dimension-sensitive cut law). *For Definition , let* $p_r = \mathrm{Tr}\,\mathscr{N}_r(I_Q/d)$. *Every deterministic decoder satisfies*

$$F_{\mathrm{e}}(\mathscr{D}\mathscr{N}) \le \frac{1}{d}\sum_r p_r \min(k_r, d),$$

$$d_{\diamond}(\mathscr{D}\mathscr{N}, \mathrm{id}_Q) \ge 1 - \frac{1}{d}\sum_r p_r \min(k_r, d).$$

In particular, if $k_r \le k \le d$,

$$F_{\mathrm{e}}(\mathscr{D}\mathscr{N}) \le k/d, \qquad d_{\diamond}(\mathscr{D}\mathscr{N}, \mathrm{id}_Q) \ge 1 - k/d.$$

For every d, k, *a finite coordinate-subset instrument and decoder attain both uniform constants.*

Proof. Write encoder Kraus maps $A_{rj} : Q \to K_r$ and branch decoder Kraus maps $B_{r\ell} : K_r \to Q'$. The composite Kraus maps $L_{r\ell j} = B_{r\ell} A_{rj}$ have rank at most $\min(k_r, d)$. The singular-value inequality gives

$$|\mathrm{Tr}\, L|^2 \le \mathrm{rank}\,(L)\,\mathrm{Tr}\,(L^\dagger L).$$

Since the branch decoder is trace preserving,

$$\sum_{\ell,j} \mathrm{Tr}\,(L_{r\ell j}^\dagger L_{r\ell j}) = \sum_{j} \mathrm{Tr}\,(A_{rj}^\dagger A_{rj}) = d p_r.$$

Use and .

For attainment, let S range over all k-element subsets of a fixed d-element basis. Let $V_S : \mathbb{C}^k \to \mathbb{C}^d$ be the coordinate inclusion, $P_S = V_S V_S^\dagger$, and

$$w = \tfrac{d-1}{k-1}^{-1}, \qquad A_S = \sqrt{w}\, V_S^\dagger.$$

Each basis coordinate belongs to $\frac{d-1}{k-1}$ subsets, so $\sum_S A_S^\dagger A_S = I$. Return S and $A_S X A_S^\dagger$. The branch decoder $Y \mapsto V_S Y V_S^\dagger$ gives

$$\Psi_k(X) = w \sum_S P_S X P_S = \lambda X + (1-\lambda)\Delta(X), \qquad \lambda = \frac{k-1}{d-1}.$$

The equality follows by counting subsets containing one coordinate and subsets containing two different coordinates. Thus $F_{\mathrm{e}}(\Psi_k) = k/d$ and

$$d_\diamond(\Psi_k, \mathrm{id}) \le (1-\lambda)\, d_\diamond(\Delta, \mathrm{id}) = 1 - k/d.$$

The Bell lower bound proves equality.

par

Immediate scope. The number k prices returned quantum access, not installed apparatus size or total working memory. The weighted bound is not a claim that every specified branch list or instrument attains it. The explicit construction proves sharpness of the uniform contract.

The Schmidt-number formulation in I-C-Ex and I-C-Ex2 yields the same bounds [7, 8, 42]. The coordinate-subset construction above avoids an unstated continuous-outcome implementation. It is finite exact attainment, not efficient physical gate synthesis.

Replacement is a stronger cut

Theorem (Exact spectral replacement law). *Let* $\mathscr{R}_\sigma(X) = \mathrm{Tr}\,(X)\sigma$ *on* $Q \simeq \mathbb{C}^d$*, and let* λ_i *be the eigenvalues of* σ*. With*

$$S_\sigma = \sum_{i=1}^{d} \lambda_i^{-1}, \qquad 1/0 = \infty, \quad 1/\infty = 0,$$

the exact fixed-channel error is

$$d_\diamond(\mathscr{R}_\sigma, \mathrm{id}_Q) = 1 - \frac{1}{S_\sigma}.$$

After any input-independent accessible channel, optimization over decoders gives

$$\varepsilon^\star = 1 - \frac{1}{d^2},$$

attained by the decoded replacement state I_Q/d*, uniquely as a replacement state. Every replacement has Bell fidelity* $1/d^2$.

Proof. Assume $\sigma > 0$. Use its eigenbasis e_i and an orthonormal reference basis f_i. For

$$|\psi_p\rangle = \sum_i \sqrt{p_i}|f_i\rangle|e_i\rangle, \qquad p_i = \lambda_i^{-1}/S_\sigma,$$

the replacement output is $\rho_E \otimes \sigma$, where $\rho_E = \sum_i p_i|f_i\rangle\mathrm{langle}f_i|$. Its overlap with the input is $\sum_i p_i^2\lambda_i = 1/S_\sigma$. The input projector gives the lower bound in .

For the upper bound put $|\Omega\rangle = \sum_i |i\rangle|i\rangle$, an unnormalized vector. Weighted Cauchy–Schwarz gives

$$I \otimes \sigma \geq \frac{1}{S_\sigma}\mathrm{proj}\,\Omega.$$

These are the Choi matrices of $\mathscr{R}_\sigma$ and id_Q/S_σ. Hence, with $s = 1/S_\sigma$,

$$\mathscr{R}_\sigma = \mathrm{sid}_Q + (1-s)\mathscr{Q}$$

for a CPTP map $\mathscr{Q}$. Two channels have half-diamond distance at most one, proving the matching upper bound. If a λ_i is zero, the input $|e_i\rangle\mathrm{langlee}_i|$ is orthogonal to σ, giving distance one.

A decoder of an input-independent output produces a replacement. Cauchy–Schwarz gives

$$d^2 \le \Big(\sum_i \lambda_i\Big)\Big(\sum_i \lambda_i^{-1}\Big) = S_\sigma,$$

with equality precisely at $\sigma = I/d$. A decoder can prepare I/d, so the optimum follows.

On Bell input the replacement output is $I_E/d \otimes \sigma$. Its Bell overlap is $\mathrm{Tr}\,\sigma/d^2 = 1/d^2$.

par

Immediate scope. The inverse-eigenvalue law concerns the *decoded replacement state*, not an unspecified bath or cosmological state. The exact fixed-state law and the optimized replacement floor are different statements.

In particular,

$$1 - \frac{1}{d} \quad \text{and} \quad 1 - \frac{1}{d^2}$$

are distinct. A generic classical channel can convey classical information about the input; a replacement conveys none. This is the comparison with I-C-CERT's thm:replacement and Big01-A's thm:Bexact [9, 19].

Complete implementation allowances

Theorem (Degradation, stability, and accumulated errors). *If $\mathscr{N}_1 = \mathscr{L}\mathscr{N}_2$ for a CPTP map $\mathscr{L}$, then*

$$\varepsilon^\star(\mathscr{N}_1) \ge \varepsilon^\star(\mathscr{N}_2).$$

For channels on the same complete interface with $d_\diamond(\widehat{\mathscr{N}},\mathscr{N}) \le \delta$,

$$|\varepsilon^\star(\widehat{\mathscr{N}}) - \varepsilon^\star(\mathscr{N})| \le \delta, \qquad |F_{\mathrm{e}}^\star(\widehat{\mathscr{N}}) - F_{\mathrm{e}}^\star(\mathscr{N})| \le \delta.$$

Accordingly a cut floor b gives implemented error at least $[b-\delta]_+$. Compatible complete stage allowances δ_j add

to at most $\min\{1, \sum_j \delta_j\}$.

Proof. Every decoder after $\mathcal{N}_1$ composes with $\mathscr{L}$ to give a decoder after $\mathcal{N}_2$, proving degradation monotonicity. For a fixed decoder, contractivity and the triangle inequality give the error difference bound. The Bell-overlap difference is bounded by the trace distance of the referenced outputs, hence by δ. Taking infima or suprema proves stability. Replace one stage at a time in a compatible composition and apply contractivity to each difference.

par

Dropping a residual is degradation, not a harmless rewriting. A record-only error estimate cannot price a decoder that still receives that residual.

Two exact counterexamples to marginal-only reasoning

Theorem (Quantum residuals and classical records can recover jointly). *There are finite channels with an input-independent classical record and exact whole-code recovery from the record together with a quantum residual.*

More strongly, there is such a channel for which both individual marginals are input independent. A replacement channel has the same individual marginals but no exact recovery.

Proof. For the first assertion take a qubit input ψ, append $|+\rangle$, apply controlled-Z, and destructively measure the input factor in the X basis. With outcome $s \in \{0, 1\}$, the remaining qubit has subnormalized state

$$\frac{1}{2}(X^s H)\mathrm{proj}\,\psi(HX^s).$$

This follows from

$$\mathrm{CZ}(\alpha|0\rangle + \beta|1\rangle)|+\rangle = \alpha|0\rangle|+\rangle + \beta|1\rangle|-\rangle.$$

Each outcome has probability 1/2. The record-controlled inverse HX^s recovers the input, with every reference. The complete output is classical–quantum, not classical-only.

For the stronger assertion use the d^2 Weyl unitaries $\{W_j\}$ and the channel

$$\mathscr{R}(X) = \frac{1}{d^2}\sum_j \mathrm{proj}\, j_K \otimes W_j X W_j^\dagger.$$

Weyl averaging gives

$$\mathrm{Tr}_K\, \mathscr{R}(X) = \mathrm{Tr}\,(X) I/d, \qquad \mathrm{Tr}_A\, \mathscr{R}(X) = \mathrm{Tr}\,(X) I_K/d^2.$$

The controlled inverse $W_j^\dagger$ recovers X exactly. The joint replacement

$$X \mathrm{longmapsto}\, \mathrm{Tr}\,(X)(I_K/d^2 \otimes I_A/d)$$

has those same two marginals. By Theorem it cannot return an unknown code.

par

The first construction is a finite measurement-based computation example with an unmeasured quantum output. The second is the private-quantum-channel mechanism [43], used as an access counterexample rather than a gravitational model. They explain why a heat-looking, random, or thermal marginal does not certify a complete classical cut [6, 7, 8].

Complete exterior, entropy, thermal accounts, and endpoints

Adaptive classical histories, including heat labels

Theorem (Insufficiency of a complete classical exterior). *A finite adaptive process with the complete classical exterior of Definition obeys*

$$\varepsilon^\star(\mathscr{C}_{\mathrm{ext}}) \geq 1 - \frac{1}{d}.$$

This includes all joint finite outcome, timing, stopping, failure, control, and heat-meter records and their correlations.

The bound persists for a diamond-norm limit of decoded finite-prefix channels on the same fixed code, when that limit exists. A complete implementation allowance δ gives the lower bound $[1 - 1/d - \delta]_+$.

Proof. Compose the subnormalized branch maps along each full terminal history h. Adaptive classical choices determine those maps but do not alter their complete positivity. Summing complete terminal branches gives trace preservation. The history probability is

$$p_h(\rho) = \mathrm{Tr}\,(E_h\rho), \qquad E_h = \mathscr{I}_h^*(I), \qquad E_h \geq 0, \quad \sum_h E_h = I.$$

Thus the complete exterior is one POVM channel. Apply Theorem ; all classical postprocessing is absorbed into the decoder.

For a limiting decoded channel, the normalized Choi states of the finite decoded prefixes are separable across $E : Q'$. They converge in trace norm. In finite dimensions the separable-state set is compact: it is the convex hull of a compact set of product states, and affine dependence reduces combinations to a uniformly bounded number of terms. The limit is separable. Its Bell overlap remains at most $1/d$, proving the floor. The allowance follows from Theorem .

par

Thermality is not a premise of this obstruction. A nonthermal classical exterior has the same bound. A full quantum reservoir or receiver memory is a different access object. A finite meter reading does not grant the meter's full quantum reservoir to the decoder.

The theorem does not price an unspecified continuum measurement, infinite transcript without a stated limit, or an acquisition map that has not been constructed.

Append-only entropy is a record theorem

Let R_n denote a complete classical history through step n. All logarithms in entropy formulas are natural. Shannon entropy is H_{cl}; von Neumann entropy is S.

Theorem (Append-only classical entropy). *If the old history is retained and the next outcome is appended, then*

$$H_{\mathrm{cl}}(R_{n+1}) = H_{\mathrm{cl}}(R_n) + H_{\mathrm{cl}}(Y_{n+1} \mid R_n) \geq H_{\mathrm{cl}}(R_n).$$

This does not imply monotonicity of the entropy of the whole classical–quantum output, a selected branch, or a coarse-grained record.

There are processes with arbitrarily long increasing append-only record entropy whose accessible channel on the unknown code is replacement throughout.

Proof. Write $R_{n+1} = (R_n, Y_{n+1})$. The classical chain rule gives the equality, and conditional classical entropy is nonnegative.

For a classical–quantum state $\omega_{RA} = \sum_r p_r \mathrm{proj}\, r \otimes \rho_{A,r}$, block diagonalization gives

$$S(\omega_{RA}) = H_{\mathrm{cl}}(p) + \sum_r p_r S(\rho_{A,r}).$$

A later operation can reduce the conditional quantum term. For instance, replacing an initially maximally mixed qubit by a fixed pure state while appending a deterministic word decreases the total accessible entropy by $\log 2$, without decreasing the old history. Discarding part of a classical history or selecting an outcome also changes the chain-rule task.

Finally discard the unknown code, keep a fixed accessible state, and append n independent fair bits. Then $H_{\mathrm{cl}}(R_n) = n\log 2$, while the source-to-accessible channel is input independent for every n.

par

This is the append-only comparison with I-C's thm:B. It is not an entropy history of gravitational radiation, a Page curve, or a proof of the thermodynamic second law. Lifetime fresh allocation, installed memory, peak live quantum space, and append-only classical storage are separate resource ledgers.

What a prefix does and does not select

Definition (Prefix and admissible continuations). A finite prefix instrument is a complete channel

$$\mathscr{P}\colon Q \to R \otimes A.$$

A continuation acts after the prefix and preserves the already recorded classical history. An *ownership selection*

specifies which quantum outputs at the terminal cut are accessible to a designated receiver.

The continuation class used in the next theorem admits: holding A, adding a fresh blank $B \simeq A$, and a local SWAP between A and B. Energy and transport restrictions, when asserted, must be stated separately.

Theorem (Prefix non-selection and the residual-information caveat). *For a prefix with a residual factor* $\dim A \geq 2$*, the continuation class of Definition contains two terminal ownership assignments: one keeps the residual on A, and another transfers it to B and leaves A blank. They agree on the entire prefix channel, its reference extension, and its old classical record.*

If the prefix carries an exactly recoverable logical subcode in A, these continuations can have different terminal recovery errors. Residual dimension at least two alone does not imply that such a logical subcode survives. Restricting the old exterior record to any deterministic subset or coarse-graining does not remove the non-selection.

Proof. Append B in a fixed pure state $|0\rangle$. The holding continuation leaves A unchanged. The SWAP continuation transfers the complete A state, including all correlations with R and the testing reference, to B. Declare B inaccessible to the receiver. The old R reduction is unchanged by either local trace-preserving continuation. The prefix is identical because no continuation acts before its end.

For an explicit recovery witness take

$$\mathscr{R}(X) = \omega_R \otimes JXJ^\dagger,$$

where $J : Q \to A$ is isometric. The holding continuation permits exact recovery. After transfer away, the receiver's output is $\mathrm{Tr}\,(X)\omega_R \otimes \mathrm{proj}\,0_A$, a replacement, with optimized error $1 - 1/d^2$.

Conversely the prefix $\mathscr{R}(X) = \mathrm{Tr}\,(X)\omega_R \otimes \mathrm{proj}\,0_A$ has the same residual dimension and preserves no unknown logical code. Thus dimension is not a sufficiency criterion for recoverability. Applying any common classical restriction to R preserves the old-record agreement.

par

The theorem says that a prefix does not *determine* an endpoint in the declared continuation class. It does not assert that every physical dynamics admits those continuations or that every residual carries useful information. If the Hamiltonian, terminal boundary condition, or allowed routing fixes the continuation, that is additional supplied data.

Both finite continuations can be assigned zero Hamiltonians and constant heat labels in an abstract local model. That example establishes no physical heat cost or gravitational endpoint. It also illustrates why the dimension qualification must not be silently strengthened to information survival.

Complete complements constrain global information

Theorem (Exact recovery, complete complement, and dilation uniqueness). *Let* $V : Q \to A \otimes B$ *be a finite isometry and define*

$$\mathscr{N} = \mathrm{Tr}_B \mathrm{Ad}_V, \qquad \mathscr{E} = \mathrm{Tr}_A \mathrm{Ad}_V.$$

All discarded quantum degrees of freedom and purifiers of mixed fixed ancillas are included in B. Then:

1. *Exact whole-code recovery from A is equivalent to*

$$\mathscr{E}(X) = \mathrm{Tr}\,(X)\sigma_B \qquad (X \in \mathscr{B}(Q)).$$

If $s = \mathrm{rank}\,\sigma_B$, *then*

$$\mathscr{N}(X) = J(X \otimes \sigma_S)J^\dagger, \qquad J : Q \otimes \mathbb{C}^s \to A \text{isometric}, \qquad \dim A \geq ds.$$

2. *The two complementary outputs cannot independently recover a nontrivial whole code.*

3. *Pure fixed-ancilla dilations of the same visible channel have complete complementary channels related by an isometry on their effective environment supports. Consequently their optimized distances to replacement agree.*

4. *Access to the full isometric output* $VXV^\dagger$ *permits a CPTP left inverse, even when a restricted classical exterior does not.*

Proof. Assume the complement is constant. Diagonalize $\sigma_B = \sum_{a=1}^{s} p_a |b_a\rangle\mathrm{langleb}_a|$, $p_a > 0$, and write

$$V|i\rangle = \sum_a \sqrt{p}_a |u_{ia}\rangle |b_a\rangle.$$

The complementary matrix-unit identities give $\langle u_{jb}|u_{ia}\rangle = \delta_{ij}\delta_{ab}$. Thus $J(|i\rangle|a\rangle) = |u_{ia}\rangle$ is isometric and gives the stated normal form. A decoder, defined on the entire A algebra, is

$$\mathscr{D}(Y) = \mathrm{Tr}_S\,(J^\dagger Y J) + \mathrm{Tr}\,[(I - JJ^\dagger)Y]\tau_Q,$$

for any fixed state τ_Q.

Conversely dilate an exact decoder by an isometry $L : A \to Q \otimes G$. Purity of the decoded basis inputs implies

$$(L \otimes I_B)V|i\rangle = |i\rangle|\xi_i\rangle.$$

Exactness on every matrix unit $|i\rangle\langle j|$ gives $\langle\xi_j|\xi_i\rangle = 1$; the vectors coincide. Tracing QG makes the complement constant. Interchanging A, B proves the second assertion.

For the third assertion, in each environment take the span of vectors $(\langle a| \otimes I)V|q\rangle$. Their Gram matrices are determined by matrix elements of the common visible channel on $|q\rangle\mathrm{langleq}'|$. Equal Gram matrices give an isometry between the two spans that intertwines the dilations. Embedding a replacement state proves one inequality for its optimized replacement distance. The CPTP left inverse

$$Y\mathrm{longmapsto}\, W^\dagger Y W + \mathrm{Tr}\,[(I - WW^\dagger)Y]\tau$$

of that isometry and contractivity prove the reverse inequality.

Finally, with $P = VV^\dagger$,

$$\mathscr{R}(Y) = V^\dagger Y V + \mathrm{Tr}\,[(I - P)Y]\tau_Q$$

is CPTP and is a left inverse of the full isometric output.

par

This is the full-code comparison with I-C-Ex and I-C-Th [7, 8, 10, 11], consistent with exact quantum error correction and no-hiding structure [44, 45]. A subsystem recovery criterion is a different statement.

In particular, the visible channel and its *complete* pure-dilation complement are not arbitrary independent information ledgers. Selected bath marginals can miss information. Heat can nevertheless vary when implementation Hamiltonians or independent resources change, as shown below.

A separate thermal ledger

Supply finite memory M, finite bath B, a bath Hamiltonian H_B, a temperature $\theta > 0$, and

$$\beta = \frac{1}{k_{\mathrm{B}}\theta}, \qquad \tau_B = \frac{e^{-\beta H_B}}{\mathrm{Tr}\, e^{-\beta H_B}}.$$

Assume the initial state is the product $\rho_M \otimes \tau_B$, and the complete memory–bath evolution is a unitary U. The final state is $\omega'_{MB} = U(\rho_M \otimes \tau_B)U^{\dagger}$. Define

$$Q_B = \mathrm{Tr}\,[H_B(\omega'_B - \tau_B)], \qquad \Delta S_M = S(\rho_M) - S(\omega'_M).$$

Positive Q_B is an increase of the bath's mean energy. It need not equal total work. This is a supplied thermal process, not a consequence of the channel name "Enter."

Theorem (Finite Gibbs-reservoir identity and reset qualification). *Under the preceding thermal assumptions,*

$$\beta Q_B = \Delta S_M + I(M : B)_{\omega'} + D(\omega'_B \| \tau_B) \geq \Delta S_M,$$

where D is quantum relative entropy and entropy uses natural logarithms.

If a degenerate classical memory with distribution p is reset to a fixed pure blank under this contract, its formal entropy decrease is $H_{\mathrm{cl}}(p)$*, so*

$$Q_B \geq k_{\mathrm{B}}\theta H_{\mathrm{cl}}(p).$$

However, an initially mixed memory cannot be reset exactly to a pure state by a unitary using only a finite full-rank Gibbs reservoir. Thus the entropy bound is not an assertion of finite-bath exact attainment.

Initial correlations, accessible side information, additional entropy sinks, changing bath models, and approximate reset require their own contracts.

Proof. Since $\log \tau_B = -\beta H_B - \log Z$,

$$D(\omega_B^{'} \| \tau_B) = \beta Q_B - [S(\omega_B^{'}) - S(\tau_B)].$$

Unitarity and the initial product give

$$I(M : B)_{\omega^{'}} = S(\omega_M^{'}) + S(\omega_B^{'}) - S(\rho_M) - S(\tau_B).$$

Combining the two equalities proves .

For completeness, relative entropy is nonnegative in finite dimensions. Diagonalize a density matrix ρ with eigenvalues r_i. Concavity of the scalar logarithm in the spectral decomposition of σ gives $\langle i|\log \sigma|i\rangle \le \log \langle i|\sigma|i\rangle$. Thus $D(\rho\|\sigma)$ is bounded below by a classical relative entropy, which is nonnegative by $\log x \le x - 1$. Support failure gives infinite relative entropy. Mutual information is $D(\omega^{'} \| \omega_M^{'} \otimes \omega_B^{'})$, hence nonnegative.

A diagonal classical memory has entropy $H_{\mathrm{cl}}(p)$; a pure blank has entropy zero. For the finite-bath obstruction, the initial global rank is $\operatorname{rank}(\rho_M)\dim B$, because τ_B is full rank. A state with a pure memory marginal is supported on $|0\rangle_M \otimes H_B$, of dimension $\dim B$. Unitary rank preservation therefore requires $\operatorname{rank}(\rho_M) \le 1$.

par

This is the separate-account comparison with I-C's thm:C and I-C-Th's thermal/reset statements [6, 10, 11]. The finite-reservoir identity is the standard equality form of Landauer's principle [46].

For a uniform m-symbol memory the formal reset bound is $k_{\mathrm{B}}\theta \log m$. It is not the readout cost of one unknown code, a cost per consumed name, or a derived horizon temperature. If a copy of the classical memory is retained as usable side information, the resetting task changes. Keeping all initial and final correlations is indispensable.

Theorem (Same visible channel, different heat bills). *A visible finite channel, its optimized recovery error, and its complete pure-dilation information structure do not determine a unique reservoir heat bill.*

At fixed $\beta > 0$, one may append an input-independent Gibbs qubit and apply an independently driven flip without changing the payload channel. For level gap $E > 0$, the additional bath-energy increase is

$$Q_{\mathrm{add}} = E\tanh(\beta E/2) > 0.$$

It can take arbitrarily large values as the supplied gap varies. Adding the corresponding independent finite classical heat report does not improve or worsen optimized payload recovery.

Adding a constant $E_0 I$ to a fixed bath Hamiltonian leaves its Gibbs state and every mean energy difference unchanged.

Proof. Append a qubit with Hamiltonian $H_{\mathrm{add}} = E\,\mathrm{proj}\,1$ in its Gibbs state. Its initial excitation probability is $p_1 = (1 + e^{\beta E})^{-1}$. An external-drive flip exchanges the two populations, giving

$$Q_{\mathrm{add}} = E[(1 - p_1) - p_1] = E\tanh(\beta E/2).$$

The operation is independent of the code and acts on a separate factor, so the payload channel is unchanged. The quantity is continuous, positive for $E > 0$, and unbounded.

An independent report state ω_F changes the accessible channel to $\mathcal{N}(X) \otimes \omega_F$. Every decoder of this output induces the decoder $Y \mapsto \mathcal{D}(Y \otimes \omega_F)$ of $\mathcal{N}$. Conversely the added report can be ignored. The optimized recovery values therefore coincide.

For $H_B^{'} = H_B + E_0 I$, the multiplicative factor $e^{-\beta E_0}$ cancels from the Gibbs normalization. Moreover

$\mathrm{Tr}\,[E_0 I(\omega_B^{'} - \tau_B)] = 0.$

par

The flip's energy is supplied by external driving; the theorem does not identify that work with a passive spontaneous heat process. It varies an implementation ledger, not the complete complementary leakage of one fixed pure dilation. This is the precise underdetermination parallel to I-C-Th [10, 11].

A finite heat reading may still be informative about an independently calibrated Hamiltonian or energy gap. The

theorem denies a channel-only universal heat bill, not the possibility of energy spectroscopy.

Conditional Big-series comparison on a supplied classical host

Reception, not a bidirectional prohibition

In this section supply $H > 0$ and the separate classical metric

$$g_H = -c^2\mathrm{mathop\,d}t^2 + e^{2Ht}\mathrm{mathop\,d}\mathbf{x}^2, \qquad \gamma(t) = (t, 0).$$

No quantum state on this host is constructed. It is not the reporting cone or either full I-D dynamical host. Put $\ell = c/H$.

Theorem (Observer reception region and fixed horizon area). *A signal from* $(t, \mathbf{x})$ *reaches* γ *at some finite future time exactly when*

$$|\mathbf{x}| < \ell e^{-Ht}.$$

The boundary is null, and every fixed-t cut has proper radius ℓ *and area*

$$A_H = 4\pi c^2/H^2.$$

These statements concern the supplied classical metric only.

Proof. Future-causal routes obey $|\dot{\mathbf{x}}| \le ce^{-Ht}$; radial null travel attains the integral bound, and the union over finite reception times yields the strict reception region. The boundary $e^{Ht}|\mathbf{x}| = \ell$ is null, and each fixed-t cut has proper radius ℓ and area $4\pi c^2/H^2$. Named parallel (style only): Big01-A [19]; the long parallel development is not re-expanded.

par

"No reception from beyond the event horizon" is observer-directed, not a bidirectional barrier. Finite area alone does not bound the number of worldline operations. No hardware energy or rate law has been supplied.

The horizon-to-channel contract and its necessary correction

A *fresh logical input* is introduced at a declared source event. It may be entangled with an untouched testing reference. Other initial resources are fixed independently of that logical input; no old receiver encoding of it is granted. Every interaction, resource dependency, quantum wire, and classical control in the complete finite network is localized and causally routed on the supplied host.

Theorem (Horizon exclusion, replacement, and classical exterior). *Under the complete fresh-input causal-network contract:*

1. *A source event outside the reception region of Theorem induces an input-independent channel to every finite receiver output on γ. Its optimized recovery error is exactly* $1 - 1/d^2$. *For a fixed decoded output state its exact error is .*

2. *A complete classical exterior, whether or not its source has a causal route, has error at least* $1 - 1/d$. *That is a sharp class-wide floor, not the exact error of every classical channel.*

3. *A retained quantum residual is a different contract. Its price is determined by the complete channel and its residual dimensions, not by the classical marginal alone.*

Proof. Pulling receiver observables backwards through a finite TP network restricts dependence to scheduled ancestors. A source outside the reception region of Theorem cannot be an ancestor, so the receiver channel is input-independent replacement; Theorem prices it. A complete classical exterior uses Theorem ; a retained quantum residual uses Definition and Theorem . Named parallel (style only): Big01-A thm:Bexact/cor:P5 [19]; not re-expanded.

par

Correction to the proposed wording. The exact no-route decoder floor does *not* become $1 - 1/d$ merely because its replacement output is also classical. Replacement satisfies the weaker classical floor but has the stronger optimized floor $1 - 1/d^2$. The classical floor is attained only by appropriate informative classical-channel

examples, such as fixed-basis dephasing followed by repreparation.

Thus a "match to the shredder floor" means use of the same *complete classical access class*, not equality of every horizon-induced channel with the classical sharp example. This keeps the thm:Bexact/thm:Bsharp comparison correct [19, 9].

A receiver can hold old classical records, thermal reports, or pre-shared resources without acquiring a newly introduced unknown input. An input-dependent initial receiver encoding, a hidden route, postselection, or an unlisted quantum factor violates the asserted contract. A field-theoretic observer algebra and source-acquisition map have not been supplied by this finite-network argument.

Fixed size, internal reorganization, and the coefficient slot

An internal reorganization changes a finite implementation while leaving the host, its constants, and declared label definitions fixed. Introduce a positive action-valued coefficient h_*. In accounts assigning it to the $\hbar$ slot, declare

$$\Theta_H(h_*) = \frac{h_* H}{2\pi k_{\mathrm{B}}}, \qquad \mathcal{S}_H(h_*) = \frac{k_{\mathrm{B}} A_H c^3}{4 G_{\mathrm{N}} h_*}.$$

At $h_* = \hbar$ these are conventional Gibbons–Hawking labels [47]. Here the formulas are supplied account functions. No thermal quantum state, entropy algebra, or finite host Hilbert space is derived.

Theorem (Host-label invariance and coefficient non-selection). *At fixed host constants and h_*, internal reorganization leaves $\ell, A_H, \Theta_H, \mathcal{S}_H$ unchanged. It may change the accessible quantum residual, decoding error, device waits, and heat bill.*

If all admitted report laws are independent of h_*, they do not identify h_* or either coefficient-sensitive label in . On $h_* \in [h_-, h_+]$, $0 < h_- < h_+$, the exact absolute minimax risks are

$$\frac{h_+ - h_-}{2}, \qquad \frac{H(h_+ - h_-)}{4\pi k_{\mathrm{B}}}, \qquad \frac{k_{\mathrm{B}} A_H c^3}{8 G_{\mathrm{N}}} \left(\frac{1}{h_-} - \frac{1}{h_+} \right),$$

respectively. The product

$$\Theta_H \mathcal{S}_H = \frac{c^5}{2G_{\mathrm{N}} H}$$

and the fixed-dimension cut floors are coefficient-blind.

Proof. Host labels are independent of finite implementation variables. Same-law estimators share output laws; the two-point absolute-error identity yields the displayed midpoint risks on $[h_-, h_+]$. The product $\Theta_H \mathcal{S}_H$ and the fixed-dimension cut floors are coefficient-blind. Named parallel (style only): Big01-A thm:Hcert [19]; shortened rather than re-proved at length.

par

This is a scalar data-fibre argument, not a claim that calibrated spectroscopy can never measure a coefficient-sensitive observable.

A proposed capacity label $\mathcal{S}_H/(k_{\mathrm{B}} \log 2)$ is not $\log_2 \dim H_{\mathrm{universe}}$ without an external promise. The static spatial region has arbitrarily many disjoint regular open subregions; functions supported on them give arbitrarily large orthogonal families. Finite radius alone therefore does not impose a finite mode count.

The coefficient h_*, an absolute energy offset, and the missing inherited characteristic graph coefficient are different slots. None is identified by a dimensionless access certificate.

Vacuum and residual-source fibres

A scalar target factors through a data map exactly when it is constant on every fibre of that map: necessity follows from factorization; sufficiency follows by assigning a value to each data fibre. For real interval targets the preceding midpoint risk is exact. No unrestricted midpoint-attainment claim is made for arbitrary nonconvex target spaces.

Theorem (Vacuum non-selection and inverse-source limits). *On a supplied connected smooth host, a shift*

$$T_\lambda = T_* + \lambda g, \qquad \lambda \in \mathbb{R},$$

preserves symmetry and conservation when T_* is conserved. Data laws independent of λ, including the

independent labels , do not determine λ. On a supplied interval $[\lambda_-, \lambda_+]$*, the absolute minimax risk is* $(\lambda_+ - \lambda_-)/2$*; over unrestricted λ it is infinite.*

On the classical de Sitter comparison,

$$G[g_H] = -\frac{3H^2}{c^2} g_H.$$

Hence the subfamily $T_* + c_0 g_H + c_G G[g_H]$ *depends only on* $c_0 - 3H^2 c_G/c^2$*, leaving a coefficient fibre even if that scalar is supplied exactly.*

For supplied $g, \Lambda, \chi \neq 0, T_{\text{known}}$*,*

$$T_{\text{res}} = \chi^{-1}(G[g] + \Lambda g) - T_{\text{known}}$$

is the unique tensor in the corresponding source-account identity. It is separately conserved exactly when T_{known} *is. At fixed host parameters the residual map is an affine isometry. It selects no microscopic substance, constitutive class, or expansion law.*

Proof. Metric compatibility preserves conservation under $T \mapsto T + \lambda g$. Same-law two-point risks give the interval and unrestricted minimax claims. On , $G = -3H^2 g_H/c^2$ yields the displayed coefficient fibre; algebraic rearrangement gives residual uniqueness and the affine isometry. Named parallels (style only): Big01-A thm:vacfibre, Big-Vac-A cor:noselect, Big06-B residual ledger [19, 25, 26, 27, 24]; shortened rather than re-proved.

par

These comparisons do not calculate absolute vacuum energy or select a particle.

The sign convention can be checked independently. Adding $-\rho_0 \int \sqrt{|g|}\,\mathrm{mathop}\,\mathrm{d}^4x$ to a regulated matter action leaves normalized fixed-background matter expectations unchanged and shifts the stress by $-\rho_0 g$, under $T_{ab} = -2\,\delta S/(\sqrt{|g|}\delta g^{ab})$. If the same equation $G + \Lambda g = \chi T$ is retained, the compensating shift is

$$T \mapsto T - \rho_0 g, \qquad \Lambda \mapsto \Lambda - \chi\rho_0.$$

Equivalently $T \mapsto T + \lambda g$ requires $\Lambda \mapsto \Lambda + \chi\lambda$. No coefficient value is selected by this bookkeeping.

The thermal identity uses energy differences and a supplied reservoir. It is not an absolute vacuum-energy calculation and does not derive the coefficient in the $\hbar$ slot.

Conditional cosmic reach and matched delivery tasks

Future conformal reach and the past-fit limitation

Supply a positive scale factor on $[t_e, \infty)$ and the separate spatially flat metric

$$\mathrm{mathop\,d}s^2 = -c^2\mathrm{mathop\,d}t^2 + a(t)^2\mathrm{mathop\,d}\mathbf{x}^2, \qquad \mathbf{x} \in \mathbb{R}^3.$$

Define comoving reaches and the proper emission-slice radius

$$\chi_T(t_e) = c\int_{t_e}^{T} \frac{\mathrm{mathop\,d}t}{a(t)}, \qquad \chi_\infty(t_e) = c\int_{t_e}^{\infty} \frac{\mathrm{mathop\,d}t}{a(t)}, \qquad D_{\mathrm{reach}}(t_e) = a(t_e)\chi_\infty(t_e).$$

Theorem (Reach and smooth past-only non-identification). *A comoving source at radius r can signal to the central observer by T exactly when $r \le \chi_T(t_e)$. Finite-time reception is possible exactly when $r < \chi_\infty(t_e)$, with the natural interpretation for an infinite integral.*

There exist positive smooth strictly expanding scale factors with identical entire smooth pasts and identical junction jets, but with finite and infinite future conformal reach. A past-only experiment whose complete conditional laws agree on these models cannot distinguish their future-horizon outcomes with equal-prior success above 1/2.

Proof. Radial null propagation and positivity of $1/a$ give the reach criteria. A smooth cutoff joining an exponential past to a late linear expansion matches all jets at a junction while sending χ_∞ to infinity; the pure exponential continuation keeps χ_∞ finite. Identical past laws force equal-prior success 1/2. The long parallel development is held as style by Series-I-IB-Enter-A and Series-II2VI-Enter-A [28, 29, 30]; it is not re-expanded here.

par

Immediate interpretation. Isolation requires a future-model assumption, not merely a past fit. An event horizon uses χ_∞; a finite deadline uses χ_T; a Hubble radius $c/H(t)$ is not generally either. Named parallel (style only): Big01-A [19].

Uniform acceleration and permanent nonacceleration

Theorem (Sharp uniform-acceleration reach bound). *Let $a > 0$ be C^2, strictly expanding, and supplied for all $t \geq t_0$. Put $H = \dot{a}/a > 0$ and $q = -a\ddot{a}/\dot{a}^2$. If*

$$q(t) \leq -\varepsilon \quad (t \geq t_0), \qquad \varepsilon > 0,$$

then

$$D_{\text{reach}}(t_0) \leq \frac{c}{\varepsilon H(t_0)}.$$

The eternal smooth expanding class is empty for $\varepsilon > 1$. For $0 < \varepsilon \leq 1$, the bound is sharp.

Proof. The margin $\ddot{a} \geq \varepsilon \dot{a}^2/a$ integrates in the a variable to the displayed bound. The identity $\dot{H} \geq (\varepsilon - 1)H^2$ empties the class for $\varepsilon > 1$. Equality histories are the constant-q power laws ($\varepsilon < 1$) and exact exponential expansion ($\varepsilon = 1$). Named parallel (style only): Series-II2VI-Enter-A / Series-VII-Enter-A thm:accel-reach [28, 29, 30].

par

Countermodel retained. Acceleration without a uniform negative bound on q is insufficient. For $t > e$, the comparison $a(t) = t\log t$ has $\ddot{a} = 1/t > 0$, but

$$\int^{\infty} \frac{\mathrm{mathop\,d}t}{t\log t} = \infty, \qquad -q = \frac{\log t}{(\log t + 1)^2} \to 0.$$

Thus "permanent acceleration" alone does not imply a finite event horizon; the uniform margin $q \leq -\varepsilon$ does the work.

Theorem (Permanently concave expansion has infinite future reach). *If* $a > 0$ *is* C^2 *on* $[t_0, \infty)$, $a \geq 0$, *and* $\ddot{a} \leq 0$ *, then*

$$\chi_\infty(t_0) = \infty.$$

Thus the spatially flat comparison has no finite future event-horizon radius for the central comoving observer.

Proof. Concavity gives $a(t) \leq a_0 + v_0(t - t_0)$. If $v_0 > 0$ the comparison integral diverges logarithmically; if $v_0 = 0$ the scale factor is constant and the integral likewise diverges. Named parallel (style only): Series-VII-Enter-A thm:nonaccel [28, 29, 30].

par

Consolidation note. Theorems , , and retain their local labels for cross-reference. Their long parallel developments are not repeated; sibling syntheses are cited as style only, not as premises. No Friedmann or Einstein-matter equation has been derived. The co-located recovery counterexample below stays in full, because it is the local correction.

A smaller horizon does not universally price recovery

Theorem (Co-located recovery counterexample). *Horizon radius alone does not impose a strictly monotone recovery error on arbitrary finite tasks.*

For two supplied de Sitter hosts with $0 < H_1 < H_2$*, the second horizon radius is smaller, but a co-located finite identity task can have zero recovery error on both hosts. The same local cut instruments can realize the same sharp access floors on both.*

Proof. Place the input and receiver at one regular observer laboratory and supply identity transport between its input and output ports. Its complete channel is id_Q, independent of H, including every reference. Its optimized error is zero on both supplied hosts. If a positive duration is required, a declared identity storage operation on the same timelike worldline gives the same channel.

Likewise the finite instruments of Theorems and can be declared locally on either host without changing their matrices, code dimensions, or accessible factors. Their errors therefore coincide despite the different radii.

par

The counterexample uses supplied ideal local channels, not an unconstructed physical storage technology. It is sufficient to refute a radius-only mathematical implication.

An epoch comparison must match the input code, source ensemble, deadline or full future reach, assistance, acquired data, delivery criterion, and noise class. A smaller radius does not itself specify their complete channel.

A matched delivery–quality benchmark

To make the proposed bridge testable, declare a flag F and a source-to-output channel

$$\mathcal{N}_{\eta,w}(X) = \eta \mathrm{proj}\, 1_F \otimes \Lambda_w(X) + (1-\eta)\mathrm{proj}\, 0_F \otimes \mathrm{Tr}\,(X)\omega,$$

where $0 \le \eta \le 1$, ω is fixed, and

$$\Lambda_w(X) = \sum_{j=0}^{d^2-1} w_j W_j X W_j^{\dagger}, \qquad w_j \ge 0, \quad \sum_j w_j = 1.$$

The Weyl unitaries satisfy $\mathrm{Tr}\,(W_j^{\dagger} W_\ell) = d\delta_{j\ell}$. The delivery flag is explicitly input independent. The loss branch is replacement, not a normalized filtered quantum branch.

Theorem (Exact flagged access and separate identifiability). *For , with* $w_{\max} = \max_j w_j$,

$$F_e^{\star}(\mathcal{N}_{\eta,w}) = \eta w_{\max} + \frac{1-\eta}{d^2}, \qquad \varepsilon^{\star}(\mathcal{N}_{\eta,w}) = 1 - \eta w_{\max} - \frac{1-\eta}{d^2}.$$

A decoder undoing a most probable Weyl unitary on delivery and preparing I/d on loss attains both optima.

When $\eta > 0$*, the flag law and delivered Bell measurements identify* η *and* w_j *separately. An unflagged channel need not identify them. At fixed w, lower delivery probability gives strictly larger error exactly when* $w_{\max} > 1/d^2$.

Proof. A decoder may be chosen separately on the two orthogonal flag blocks. For a delivery decoder with Kraus operators K_α, define

$$q_j = \frac{1}{d^2}\sum_{\alpha} |\mathrm{Tr}\,(K_\alpha W_j)|^2.$$

Orthogonality and completeness of the Weyl operator basis give

$$\sum_j q_j = \frac{1}{d}\sum_{\alpha} \mathrm{Tr}\,(K_\alpha^\dagger K_\alpha) = 1.$$

Thus $F_{\mathrm{e}}(\mathscr{D}\Lambda_w) = \sum_j w_j q_j \le w_{\max}$. Undoing a maximizing W_j attains $w_{\max}$. Every decoded loss branch is replacement and has Bell fidelity $1/d^2$. This proves the first optimum in and the distance lower bound.

Choose the stated decoder. The corrected delivery branch is another Weyl mixture with identity weight $w_{\max}$. The uniform replacement I/d is the uniform Weyl mixture. Their combined decoded channel has identity weight

$$a = \eta w_{\max} + (1 - \eta)/d^2.$$

Its Bell images under different Weyl operators are orthogonal, so the Bell test gives distance $1 - a$. The triangle inequality over the nonidentity unitary channels gives the matching upper bound.

The delivery flag probability is η on every input. On Bell input, flag 1 and Bell label j have joint probability ηw_j. For $\eta > 0$, division by the separately identified η gives w_j. At $\eta = 0$ the delivered channel is unobserved.

For an unflagged example take $\omega = I/d$ and uniform $w_j = 1/d^2$. The unflagged channel is completely depolarizing for every η; hence the decomposition is not identifiable from it. Differentiating the exact error with respect to η gives $-w_{\max} + 1/d^2$, proving the final statement.

par

This is an exact finite benchmark, not a derived cosmic delivery law. The flag must be calibrated independently: a lost detection can result from detector failure or local decoherence, not necessarily geometric exclusion. "Separate identifiability" means separate parameters in the declared model, not independent statistical estimators or zero calibration uncertainty.

All loss trials are kept in . Conditional delivered-state tomography is not a replacement for the unconditional

whole-code recovery task. If flag probabilities depend on the input, division by a branch probability generally does not define a channel, and the theorem cannot be applied unchanged.

When independent source locations, route timing, and a fixed deadline are supplied, geometry can bound or identify which routes are allowed. A source-to-apparatus construction and complete error allowance are still needed to identify the observed η with that causal delivery quantity.

Imperfect Records, Approximate Reset, and Heralded Access

Guiding principle. Imperfect flags, approximate reset, private keys, and heralded cuts have quantitative prices inside declared channel and thermal classes. The four theorems below are conditional deductions from printed assumptions; the four research conjectures ask whether physical systems realize those assumptions. No conjecture is promoted to a theorem. Named parallels to I-C, I-C-Ex/Ex2, I-C-CERT, I-C-Th, En09-B, Series-I-IB-Enter-A, Series-II2VI-Enter-A, Series-VII-Enter-A, Big01-A, Big-Vac-A, and Big06-B are style and source-record comparisons, not premises.

Imperfect flags and record refinement

Theorem prices a perfect delivery flag together with a delivered Weyl mixture. The next identity extends that benchmark to imperfect or coarse classical records inside the same declared class.

Theorem (Optimal recovery from a classically flagged Weyl mixture). *Let* $\{W_j\}_{j=0}^{d^2-1}$ *be an orthogonal Weyl unitary basis (*$\operatorname{Tr}(W_j^\dagger W_\ell) = d\delta_{j\ell}$*) and let* $p_{fj} \geq 0$ *with* $\sum_{f,j} p_{fj} = 1$*. For the classically flagged channel*

$$\mathcal{N}(X) = \sum_f \operatorname{proj} f \otimes \sum_j p_{fj} W_j X W_j^\dagger,$$

$$F_{\mathrm{e}}^\star(\mathcal{N}) = \sum_f \max_j p_{fj}, \qquad \varepsilon^\star(\mathcal{N}) = 1 - \sum_f \max_j p_{fj}.$$

An optimal decoder applies, on flag f, the inverse of a maximising Weyl operator.

Proof. A decoder may be chosen separately on orthogonal flag blocks. On flag f, decoder Kraus operators K_α give Weyl weights

$$q_{fj} = \frac{1}{d^2} \sum_{\alpha} |\mathrm{Tr}\,(K_\alpha W_j)|^2$$

with $\sum_j q_{fj} \le 1$ under trace preservation (or after discarding unused mass). Hence the contribution of flag f is at most $\max_j p_{fj}$, so

$$F_{\mathrm{e}}(\mathscr{D}\mathscr{M}) \le \sum_f \max_j p_{fj}.$$

Applying the inverse of a maximising W_j on each flag attains the bound. Averaging over flags yields identity weight $\sum_f \max_j p_{fj}$; convexity of the remaining unitary channels makes the diamond error exact and equal to one minus that weight.

par

Corollary (Exact coarse-graining penalty). *If $g = g(f)$ is any classical coarse-graining of the flag alphabet, write $p_{gj} = \sum_{f\,:\,g(f)=g} p_{fj}$. Then*

$$\varepsilon^\star_{\mathrm{coarse}} - \varepsilon^\star_{\mathrm{fine}} = \sum_f \max_j p_{fj} - \sum_g \max_j p_{gj} \ge 0.$$

Proof. For each coarse flag g, $\max_j p_{gj} = \max_j \sum_{f\,:\,g(f)=g} p_{fj} \le \sum_{f\,:\,g(f)=g} \max_j p_{fj}$. Summing over g and applying Theorem to both the fine and coarse channels gives the identity.

par

This prices record refinement inside a declared channel class. It extends Theorem from a perfect delivery label to imperfect or coarse flags.

Imperfect-flag penalty number (not a new theorem). Take $d = 2$ with Weyl labels $\{I, X, Y, Z\}$ and two flags $\{f_0, f_1\}$. Let the flagged weights be

$$p_{f_0,I} = 0.40,\ p_{f_0,X} = 0.10,\ p_{f_1,I} = 0.10,\ p_{f_1,Z} = 0.40,$$

and zero otherwise. Theorem gives

$$F^\star_{\mathrm{e}} = \max(0.40, 0.10) + \max(0.10, 0.40) = 0.80, \qquad \varepsilon^\star = 0.20.$$

Coarse-grain the two flags into one record. Then $p_I = 0.50$, $p_X = 0.10$, $p_Z = 0.40$, so $F^{\star}_{\mathrm{e,\,coarse}} = \max = 0.50$ and $E^{\star}_{\mathrm{coarse}} = 0.50$. The exact coarse-graining penalty of Corollary is the concrete number

$$\varepsilon^{\star}_{\mathrm{coarse}} - \varepsilon^{\star}_{\mathrm{fine}} = 0.50 - 0.20 = 0.30.$$

Refining the flag therefore recovers thirty percentage points of diamond error inside the declared Weyl-mixture class. The number is an audit of the imperfect-flag theorem; it is not a horizon model and does not open programme Host-A. A second imperfect-flag packet on the same Weyl alphabet uses three flags with weights $p_{f_0,I} = 0.50$, $p_{f_1,X} = 0.30$, $p_{f_2,Y} = 0.20$. Then $F^{\star}_{\mathrm{e}} = 1.00$ and $\varepsilon^{\star} = 0$: each flag identifies a unique Weyl operator, so the channel is perfectly correctable inside the class. Coarse-graining all three flags into one record collapses the weights to $p_I = 0.50$, $p_X = 0.30$, $p_Y = 0.20$, hence $F^{\star}_{\mathrm{e,\,coarse}} = 0.50$ and a penalty of 0.50. The contrast between the first packet (penalty 0.30) and this second packet (penalty 0.50) is the numerical content of Corollary : the penalty equals the drop in $\sum \max_j p_{\cdot j}$ under the chosen coarse-graining, and nothing else. Parent-table notes that the I-C-Ex / I-C-Ex2 and I-C-Th / I-C-Th-CERT records are source comparisons, not premises, belong in the parent-comparison table rather than in any bibliography entry; the present arithmetic does not disturb that discipline. It is *not* a horizon model, a Gibbons–Hawking identification, or a derived cosmic delivery law. Series-I-IB-Enter-A lem:pure-branch-marginal (style only) reminds that a pure positive branch need not yield a pure subsystem marginal when an untouched reference is kept; that separation is not used as a premise here.

Approximate reset under spectral constraints

Theorem and the named I-C-Th thm:reset record a rank obstruction to exact pure reset of a mixed memory by a finite full-rank reservoir alone. The next bound prices the minimum error under unrestricted joint unitaries.

Theorem (Minimum reset error under unrestricted joint unitaries). *Let $\rho_M \otimes \tau_B$ be a product initial state on memory × bath of dimensions m and b, with joint eigenvalues $\lambda_1 \geq \cdots \geq \lambda_{mb}$, and let $|0\rangle_M$ be a fixed blank. Then*

$$\varepsilon_{\mathrm{reset}} = \inf_U \frac{1}{2} \| \mathrm{Tr}_B [U(\rho_M \otimes \tau_B)U^{\dagger}] - \mathrm{proj}\, 0_M \|_1 = 1 - \sum_{\ell=1}^{b} \lambda_{\ell}.$$

The blank subspace $H_{\text{blank}} = \text{Span}\,\{|0\rangle_M\} \otimes H_B$ *has rank b, so its maximum attainable population is the sum of the largest b eigenvalues. Mapping those eigenvectors into the blank subspace and the remainder into its complement attains the distance.*

If ρ_M *is mixed and* τ_B *is full rank, then* $\varepsilon_{\text{reset}} > 0$. *If both factors are maximally mixed, then* $\varepsilon_{\text{reset}} = 1 - 1/m$*: enlarging a maximally mixed bath alone does not help. Energy conservation, bounded work, locality, or finite time can only raise the error above .*

Proof. For any unitary U, write $\sigma = U(\rho_M \otimes \tau_B)U^\dagger$ and $\rho_M' = \text{Tr}_B\,\sigma$. The overlap of ρ_M' with $\text{proj}\,0$ equals the population of H_{blank} under σ. By the Ky Fan maximum principle that population is at most $\sum_{\ell=1}^{b} \lambda_\ell$, attained by mapping the supporting eigenvectors of the largest b eigenvalues into H_{blank}. For a pure target, $\frac{1}{2}\|\rho_M' - \text{proj}\,0\|_1 = 1 - \langle 0|\rho_M'|0\rangle$, yielding .

If ρ_M is mixed and τ_B is full rank, then $\text{rank}\,(\rho_M \otimes \tau_B) > b$, so the blank population is strictly less than 1. If both factors are maximally mixed, the sum of the largest b eigenvalues equals $1/m$. Any energy, work, locality, or timing restriction shrinks the admissible unitary class and therefore cannot decrease the infimum.

par

Named comparisons: Theorem and I-C-Th thm:reset (exact reset is a different interface). The spectral bound is the quantitative next step; it does not assert saturation under energy-conserving or local controls.

Private-key resource and key-reset heat

Theorem (Key size and entropy for exact random-unitary encryption). *Let*

$$\mathscr{P}(X) = \sum_{r=1}^{n} p_r \text{proj}\, r \otimes U_r X U_r^\dagger$$

with $p_r > 0$ *and quantum marginal exactly depolarising* ($\sum_r p_r U_r X U_r^\dagger = \text{Tr}\,(X)\, I/d$ *for every X). Then*

$$n \geq d^2, \qquad H(p) \geq 2\log d.$$

Both bounds are attained by a uniform Weyl key.

A subsequent product-memory/Gibbs-bath reset of that key, with no usable side information, has formal heat

$$Q_B \geq 2k_{\mathrm{B}}\theta \log d.$$

That is a key-reset bound, not the cost of encryption or readout. The finite-bath obstruction of Theorem still excludes exact reset of this mixed key by a finite full-rank reservoir alone.

Proof. On Bell input the depolarising condition produces a full-rank mixture of n rank-one states on a d^2 -dimensional space. Each weight is at most $1/d^2$ (otherwise the Choi operator could not be the maximally mixed state of rank d^2). Hence $n \geq d^2$, and Shannon entropy satisfies $H(p) \geq \log(d^2) = 2\log d$, with equality for the uniform distribution. A uniform Weyl key attains both bounds by the standard private-channel construction.

For the subsequent reset, take the classical key register as the memory to be returned to a blank, with Gibbs bath at inverse temperature $1/(k_{\mathrm{B}}\theta)$ and no side information about the key. Theorem 's thermal identity and the entropy lower bound $H(p) \geq 2\log d$ give the formal heat lower bound $Q_B \geq 2k_{\mathrm{B}}\theta \log d$ under the stated product and unitary hypotheses. Exact pure reset of the mixed key remains forbidden by the rank obstruction when the reservoir is finite and full rank.

par

Remark (key-reset heat is not universal). The formal bound $Q_B \geq 2k_{\mathrm{B}}\theta \log d$ holds under the product-memory/Gibbs-bath hypotheses of Theorem . Theorem still shows that a different Hamiltonian constant or a different reservoir model can produce the same visible channel with a different heat bill. No universal heat-per-qubit law is claimed.

Named parallel: the Weyl-key counterexample already in the debug ledger; Theorem is the append-only entropy comparison, not a Page curve. I-C thm:B is cited as a named comparison only.

Heralded return through a subdimensional cut

Theorem (No nonzero exact heralded return through a subdimensional cut). *Suppose every accessible branch of an instrument factors through a quantum space of dimension at most $k < d$, and a completely positive*

success map $\mathscr{S}$ satisfies

$$\mathscr{S}\mathscr{N} = p\,\mathrm{id}_Q$$

for an input-independent scalar p. Then p = 0.

Every success Kraus operator has rank at most k. A nonzero multiple of the identity has Choi rank one and forces every nonzero success Kraus operator to be proportional to I_Q, which has rank d.

Proof. Factorisation through a *k*-dimensional space implies that every Kraus operator of $\mathscr{S}\mathscr{N}$ (equivalently, every success Kraus operator after absorbing the instrument's Kraus operators) has rank at most *k*. If $p \neq 0$, then $\mathscr{S}\mathscr{N} = p\,\mathrm{id}_Q$ has Choi operator proportional to the maximally entangled projector, which has rank one and forces a single (up to phase) Kraus operator proportional to I_Q. But rank $(I_Q) = d > k$, a contradiction. Hence $p = 0$.

par

Scope: exact universal one-use return only. Approximate heralded recovery, a smaller subcode, and additional-copy tasks are not excluded. Named parallel: I-C-Ex2 thm:mbqc and lem:fivecode are different tasks and are not premises for this obstruction.

Research conjectures: physical bridges, not released theorems

The finite benchmarks above, including the imperfect-access identities of Section , are proved. The eight statements below concern unsupplied physical realizations. No conjecture is promoted to a theorem; none is used in any proof.

Completion-criterion table

toprule Label Test / completion criterion

midrule endhead conj:epoch Matched fresh-input tasks with fixed code/ensemble/assistance/deadline and calibrated complete receiver channel, for which a smaller-reach epoch yields a strictly tighter operational floor (not a universal radius-to-error rule; cf. Theorem).

conj:joint Independently calibrated delivery flag and quality test that separate geometric exclusion from local noise in a declared model class (finite diagnostic: Theorem).

conj:thermal Complete physical thermal ledger (acquisition, readout, reset, work, heat) with verified coefficient-blind data map; does not derive $\hbar$ or absolute vacuum energy.

conj:endpoint Matched prefix experiment admitting alternative terminal ownership assignments under one supplied continuation class and the same horizon-interior exterior data (local routing: Theorem).

conj:record-refinement Fine versus coarse calibrated error records with fixed quantum output and noise; witness Corollary under declared calibration uncertainty.

conj:spectral-reset-realization Permitted controls approach with separate work/heat/time accounts; no universal heat-per-qubit law.

conj:reservoir-access Retained quantum reservoir beats the classical-only floor of Theorem (or residual/replacement floor) on the same classical heat and outcome records (style: Series-VII-Enter-A thm:classical-imitation).

conj:recovery-reset-frontier Attainable set $\mathscr{A} = \{(\varepsilon_{\text{recovery}}, \varepsilon_{\text{reset}}, Q_B)\}$ strictly inside the product of separate channel and thermal bounds under a fixed implementation class (cf. Theorem).

bottomrule

Standing boundary. No spacetime emergence, derived metric, Einstein dynamics, Page curve, island, cosmological bounce, de Sitter quantum state, vacuum-energy selection, inherited characteristic quantum coefficient, compilation, or Host C is claimed. O.1 stays NOT_CLOSED; Gate 0 and ST4b stay NOT_SUPPLIED; XV-B is not opened; XVI-N is not discharged; $\mathrm{Dom}(T)$ remains undetermined; J2 remains unresolved; Problems 6, 7, and 9 are named, not solved. Locked Host-A stays NAMED_COMPARISON_ONLY, with $\{\mathrm{GI}, \mathrm{CB}\}$ only. Cited Cheung and OpenAI records are source records, not premises. The declared path is null; wired=false.

Short scopes (not theorems)

Research conjecture (Matched-task epoch sensitivity). Physically justified matched retrieval tasks on a supplied expansion history for which the smaller-reach epoch is operationally stricter.

Scope. Not a universal radius-to-error rule; horizons, Hubble radii, and comoving radii must not be interchanged; no inflationary state is identified.

Research conjecture (Joint geometric–quantum access). A matched experiment separately certifies causal delivery and delivered channel quality with a complete flag/failure interface.

Scope. Needs calibrated sources, clocks, routes, and reference-preserving quality tests; unflagged decay or a thermal marginal is insufficient.

Research conjecture (Thermal-ledger bridge). A controlled shredder/reset realization admits a complete thermal ledger whose coefficient-blind data map selects no absolute vacuum energy or $\hbar$-slot normalization.

Scope. Finite identity is already proved; the bridge is physical realization and verified blindness.

Research conjecture (Endpoint alternatives under a supplied horizon cut). Alternative terminal ownership assignments for a surviving residual under one supplied continuation class and the same prefix/exterior data.

Scope. Local routing is Theorem ; O.1 stays NOT_CLOSED; no gravitational endpoint is inferred.

Research conjecture (Calibrated record refinement). Calibrated error records whose inclusion strictly improves whole-code recovery as in Theorem .

Scope. Witness the coarse-graining penalty of Corollary under calibration uncertainty.

Research conjecture (Spectral-reset realization). Permitted controls approach the spectral reset bound of Theorem with accounted work/heat/time.

Scope. Unrestricted-unitary saturation does not imply energy-conserving saturation; no universal heat-per-qubit law.

Research conjecture (Reservoir-assisted access beyond classical records). A retained quantum reservoir supports recovery unavailable from complete classical heat and outcome records alone.

Scope. Access-dependent recoverability, not anomalous heat or a gravitational endpoint.

Research conjecture (Joint recovery–reset frontier). Under a fixed implementation class, the attainable $(\varepsilon_{\text{recovery}}, \varepsilon_{\text{reset}}, Q_B)$ set is stricter than the product of separate bounds.

Scope. Not a channel-only law; Theorem explains underdetermination of heat by the visible channel.

Standing boundary. No spacetime emergence, derived metric, Einstein dynamics, Page curve, island, cosmological bounce, de Sitter quantum state, vacuum-energy selection, inherited characteristic quantum coefficient, compilation, or Host C is claimed. O.1 stays NOT_CLOSED; Gate 0 and ST4b stay NOT_SUPPLIED; XV-B is not opened; XVI-N is not discharged; $\mathrm{Dom}(T)$ remains undetermined; J2 remains unresolved; Problems 6, 7, and 9 are named, not solved. Locked Host-A stays NAMED_COMPARISON_ONLY, with $\{\mathrm{GI}, \mathrm{CB}\}$ only. Cited Cheung and OpenAI records are source records, not premises. The declared path is null; wired=false.

Exact release and unchanged programme obligations

Theorem (Series-IC-Enter-A local release). *The displayed interfaces establish:*

1. *sharp classical, residual-dimension, and replacement recovery contracts, with finite attainments and complete implementation allowances;*

2. *complete classical-exterior insufficiency, append-only classical entropy, and prefix ownership non-selection in the stated continuation class;*

3. *exact-complement constraints and separate finite thermal identities and heat-bill underdetermination;*

4. *conditional observer reception and horizon-access comparisons on a separate supplied classical host;*

5. *coefficient and vacuum non-selection for the displayed data fibres, and residual-source uniqueness only after*

source supply;

6. *conditional future-reach bounds, a radius-only recovery counterexample, and an exact flagged delivery–quality benchmark;*

7. *imperfect-flag optima and coarse-graining penalties, spectral approximate-reset minima, private-key size/entropy and key-reset heat bounds, and the exact heralded-cut obstruction (Section). Eight physical bridge conjectures remain unproved; they are listed compactly in Table with short scopes, and none is promoted to a theorem. This is a finite/local documentary synthesis, not a shredder-to-horizon identification or gravitational endpoint theory.*

Proof. Theorems – prove the first item. Theorems , , and prove the second. Theorems , , and prove the third.

Theorems and use the independent prescribed metric and complete causal-network assumptions. Theorems and concern explicit account fibres, not quantum state selection. Theorems – prove the sixth item under their printed future and channel hypotheses. Theorems – and Corollary prove the seventh item under their printed channel and thermal hypotheses.

No proof uses a research conjecture. The types remain separate: a classical marginal is not a retained quantum output; a finite code is not a horizon Hilbert space; a bath heat difference is not an absolute stress normalization; a prefix is not a terminal continuation; a causal bound is not a complete physical channel; an imperfect flag is not a horizon model; key-reset heat is not an encryption cost; exact heralded return is not approximate recovery. These separations make the local assembly compatible without supplying any missing physical or analytic admission map.

par

The inherited analytic boundary is not entered

No full joint characteristic map, joint section, transverse estimate, contact compatibility theorem, or quantum output wavefront theorem for the inherited graph is supplied here. A finite report, decoder, or access certificate

cannot substitute for that chain [31, 32, 33, 34].

The historical H2 statement in XIV-H and the narrower constructed internal I2 amplitude in XIV-I must be version-qualified. Constructing an internal amplitude does not discharge the later I3/J2 joint composition. Technical statements in XV-B do not constitute an opened downstream quantum-admission exit. This manuscript does not assess a separately scoped source claim as a global programme closure.

The no-selection conclusions above are data-fibre results. The unresolved gates are missing constructions, not universal nonexistence theorems.

Standing records and external catalogue

No local cosmological quantum state is installed. The finite-Hilbert promise remains external. The source domain Dom (*T*), J2, and the Problem 9 programme retain their standing statuses [35, 36, 37, 38, 39]. Problems 6, 7, and 9 are named, not solved.

The OpenAI catalogue record is cited only as a separation [50]. No family is used to supply a unique expansion law, a microscopic source, a shredder–horizon identification, a physical clock unit, or the inherited characteristic coefficient. The catalogue's reported proof status is not a certificate of this sheet.

Submission and verification declarations

The proofs are self-contained in the stated finite matrix and prescribed-metric classes, using standard finite-channel and entropy background with appropriate references. Cheung citations identify supplied manuscript records or supplied article metadata. No independent scan or validation of the full companion corpus or its latest versions is asserted.

Successful double pdflatex typesetting under SOURCE_DATE_EPOCH=1792627200 **is** asserted for this Version 3 deliverable. Proof-assistant checks, independently executed numerical validation, registration, referee approval, and journal acceptance are **not** asserted. Authorship is recorded as "Ngo Cheung". authorship of this synthesis. Nothing is registered or pushed (wired:false; declaredPath=null).

appendix

Finite-clock normalization audit

This appendix distinguishes three quantities that are sometimes confused:

$$d_{\diamond}(\Delta_C, \mathrm{id}_C) = 1 - \frac{1}{D}, \qquad \|\Delta_C - \mathrm{id}_C\|_{\diamond} = 2 - \frac{2}{D}, \qquad c_D = 2 - \frac{2}{D}.$$

The last number in En09-B's thm:cDsharp is a sharp *state-distance ratio*, not merely the unhalved channel norm.

Let $\Delta_C(X) = \sum_j P_j X P_j$, tensored with the identity on all unread factors K. For a state ρ, define

$$e(\rho) = \frac{1}{2}\|\rho - \Delta_C\rho\|_1, \qquad b(\rho) = \inf_{\sigma = \Delta_C\sigma} \frac{1}{2}\|\rho - \sigma\|_1.$$

Theorem (Distinct sharp finite-clock quantities). *For the destructive clock with arbitrary survivor A,*

$$\inf_{\mathscr{D}} d_{\diamond}(\mathscr{D}\mathscr{M}_C, \mathrm{id}_{CA}) = 1 - \frac{1}{D}.$$

Basis re-preparation attains the optimum and is exact precisely on clock-block-diagonal inputs.

Separately,

$$b(\rho) \le e(\rho) \le (2 - 2/D)b(\rho).$$

The constant $2 - 2/D$ is sharp when the unread factor has dimension at least $2D$. For $D = 2$, $e(\rho) = b(\rho)$.

For the different supplied logical encoding

$$W\psi = D^{-1/2}\sum_j |\tau_j\rangle_C \otimes U_j\psi,$$

the clock output permits exact logical recovery by the record-controlled inverse $U_j^{\dagger}$. That does not recover arbitrary unencoded CA.

Proof. Basis re-preparation after destructive readout equals $\Delta_C \otimes \mathrm{id}_A$. The phase-unitary average gives $d_{\diamond}(\Delta_C, \mathrm{id}_C) \le 1 - 1/D$. On Bell input the dephasing defect is $D^{-1}(J - I)$, with eigenvalues $(D - 1)/D$ once and $-1/D$ $D - 1$ times. This proves equality. For arbitrary CA, the cut has input dimension $D\dim A$ and returned quantum

blocks of dimension $\dim A$. Theorem supplies the matching universal lower bound.

Every block-diagonal σ is fixed by Δ_C, so

$$\rho - \Delta_C\rho = (\mathrm{id} - \Delta_C)(\rho - \sigma).$$

The exact channel norm just computed gives $e(\rho) \le (2 - 2/D)d_{\mathrm{tr}}(\rho, \sigma)$. Minimize; the reverse inequality $b \le e$ follows by choosing $\sigma = \Delta_C\rho$.

For sharpness choose orthonormal unread vectors $|0\rangle, \ldots, |2D - 1\rangle$. Put

$$g_j = |\tau_j\rangle|j\rangle, \qquad h_j = |\tau_j\rangle|D + j\rangle,$$

and let Φ, Φ' be uniform coherent projectors on the respective D-dimensional spans. The state

$$\rho = \Delta_C\Phi' - \Phi'/D + \Phi/D$$

is positive: on the two spans its matrices are J/D^2 and $I/D - J/D^2$, respectively. It has trace one and $e(\rho) = (2 - 2/D)/D$. The choice $\sigma = \Delta_C\Phi'$ gives $b \le 1/D$.

For the reverse bound use the Hermitian contraction

$$Z = (-I + 2J/D) \text{on the} g\text{-span}, \qquad Z = (I - 2J/D) \text{on the} h\text{-span},$$

and zero elsewhere. Then $\mathrm{Tr}\, Z\rho = 1$ and $\lambda_{\max}(\Delta_C Z) = 1 - 2/D$. For every block-diagonal state σ,

$$d_{\mathrm{tr}}(\rho, \sigma) \ge \frac{1}{2}[\mathrm{Tr}\, Z\rho - \mathrm{Tr}\,(\Delta_C Z)\sigma] \ge 1/D.$$

Thus $b = 1/D$, proving sharpness. For $D = 2$, the two general inequalities coincide.

Finally $W^\dagger W = I$, and destructive clock readout gives

$$\frac{1}{D}\sum_j \mathrm{proj}\, j_R \otimes U_j X U_j^\dagger.$$

The controlled inverse followed by discarding the record returns X, including every reference. Its target is logical Q, not arbitrary CA.

par

The state-ratio theorem does not resolve smaller-unread-dimension or restricted-product-input variants left open in the source [17, 18]. The appendix corrects the attribution of thm:cDsharp; it imports no source theorem as a premise. Named comparisons (style only; not re-proved): Series-I-IB-Enter-A lem:no-ccr (finite-matrix bandwidth ≠ CCR) and En09-B thm:tradeoffdiamond (a different supplied task) [28, 29, 30, 17, 18].

Debugging ledger

toprule Proposed shortcut Corrected statement

midrule endhead A classical marginal is the complete shredder output. Only if every other assisting quantum factor is inaccessible. A retained residual changes the cut.

The classical floor is the exact error of every classical channel. It is a sharp class-wide lower bound. Individual channels can have larger errors.

Replacement and classical-only floors coincide. Their optimized floors are $1 - 1/d^2$ and $1 - 1/d$.

No horizon reception gives exactly the generic classical floor. For a fresh input and complete causal-network contract it gives replacement, hence the stronger floor.

A thermal-looking marginal proves loss of quantum information. The Weyl-key example has two constant marginals and exact joint recovery.

A short transcript excludes entanglement-assisted recovery. Only a verified complete classical factorization excludes the assisting receiver quantum resource.

Residual dimension at least two proves surviving logical information. A two-dimensional constant residual is a counterexample. An information-bearing subcode needs a separate certificate.

Prefix non-selection gives arbitrary different complete leakage for one fixed visible channel. Pure-dilation complete complements are isometrically equivalent. Future ownership changes and selected marginals are

different questions.

Append-only Shannon entropy is a Page curve or thermodynamic entropy production. It is a classical chain-rule result for one retained history.

Destructive readout automatically has a Landauer reset bill. Readout and a specified reset process are different interfaces. The thermal identity requires product initial memory and Gibbs bath, unitary evolution, and a supplied Hamiltonian.

An exact pure mixed-memory reset is attained using a finite full-rank thermal bath alone. Unitary rank preservation forbids that reset. Approximation or additional entropy sinks must be specified.

Heat determines absolute energy or stress normalization. Hamiltonian constants cancel from Gibbs states and heat differences; vacuum shifts remain invisible to the declared coefficient-blind data.

A supplied thermal account can never contain information about $\hbar$. A independently calibrated coefficient-sensitive spectrum can. Non-selection applies only to a data map verified to be blind.

A horizon is a barrier in both causal directions. The no-reception statement is observer-directed; outward rays can cross the boundary.

A finite radius or horizon area derives a finite Hilbert space. A finite region supports arbitrarily many orthogonal functions. Finite code dimension or a host promise is additional data.

A past fit fixes future horizon existence. Smooth identical pasts admit opposite future conformal integrals.

Permanent acceleration always gives a horizon. $a = t\log t$ is an accelerating counterexample. A uniform future margin supplies the bound.

Any $\varepsilon > 0$ is compatible with an eternal smooth expanding future satisfying $q \leq -\varepsilon$. The class is empty for $\varepsilon > 1$. The sharp nonempty range is $0 < \varepsilon \leq 1$.

A smaller horizon universally worsens recovery. Co-located identity transport has zero error on either host. Matched tasks and derived delivery/noise channels are necessary.

A delivered conditional state is the unconditional channel. Failure blocks must remain. Input-dependent postselection does not generally normalize to a channel.

En09-B's $c_D = 2 - 2/D$ is simply a recovery probability. It is a sharp state-distance ratio; the universal half-diamond clock-loss error is $1 - 1/D$.

A technical lifting result opens the inherited quantum-admission exit. No full graph-specific joint map or admitted coefficient is provided by this sheet.

Imperfect or coarse flags are a horizon model. Theorem prices recovery inside a declared flagged Weyl class; it is not a horizon model or Gibbons–Hawking identification.

Coarse-graining never changes the recovery floor. Corollary gives the exact nonnegative penalty $\sum_f \max_j p_{fj} - \sum_g \max_j p_{gj}$.

Approximate reset is free under energy constraints. Theorem is an unrestricted-unitary lower bound; energy conservation, work, locality, or finite time can only raise the error.

Private-key heat is the cost of encryption or readout. The bound $Q_B \geq 2k_B\theta \log d$ is a key-reset bound under product/Gibbs hypotheses, not an encryption or readout cost.

Heralded return through a cut is impossible even approximately. Theorem excludes only exact universal one-use return with $p\,\mathrm{id}_Q$; approximate recovery, smaller subcodes, and additional-copy tasks are not excluded.

A parent label is a local premise. Every cited Cheung or OpenAI label is a source record; local hypotheses are printed independently.

bottomrule

Unexecuted documentary manifest

id: Series-IC-Enter-A?alias: IC-Enter-A?version: 4?request: t393u?role: documentary-synthesis (consolidation)?declaredPath: null?wired: false?local-interface-proofs:

constructed-on-displayed-domains?classical-host-comparisons: conditional?physical-bridge-conjectures: unproved (eight labelled; compact table)?imperfect-access-section: sec:imperfect-access?parent-comparison-table: tab:parent-comparison?conjecture-table: tab:conjectures?retained-imperfect-theorems:? - thm:imperfect-flag? - thm:approximate-reset-spectrum? - thm:private-key-resource? - thm:heralded-cut?retained-corollary: cor:coarse-flag?retained-conjectures:? - conj:epoch? - conj:joint? - conj:thermal? - conj:endpoint? - conj:record-refinement? - conj:spectral-reset-realization? - conj:reservoir-access? - conj:recovery-reset-frontier?cosmic-proofs: shortened (labels retained; epoch-counterexample full)?spacetime-emergence: not-claimed?inherited-quantum-admission: not-established?Host-A:? status: NAMED_COMPARISON_ONLY? tags: [GI, CB]?PC: PC20-IC + PC21-IC?DC: DC20-IC + DC21-IC?frozen_v1_tex_sha256: 37bc0e67a15038730802366a73529f9a294677c1031f4f16e6eb00224f880c61?frozen_v2_tex_sha256: 9b3b98c337f9fab1a3e4826c6d2f81f4a9fcffc00e500fa44a93f7ef6eb8e293

No source record is changed by the manifest. Any future registration must distinguish source-local labels from this file's local labels, retain the displayed boundaries, and state its versioned source metadata explicitly.

Plan corrections PC20-IC and draft verdicts DC20-IC

toprule ID Item Status

midrule endhead PC20-IC-1 Retain StandingBoundary after every theorem (via textbackslash Boundary); manual StandingBoundary after every new conjecture; do not close O.1 / Gate0 / ST4b / Dom(T) / J2 / Problems 6/7/9; XV-B not opened; XVI-N not discharged; Host-A stays NAMED_COMPARISON_ONLY with $\{GI, CB\}$. held

PC20-IC-2 Author "Ngo Cheung"; date 28 October 2026; request t392u; Version 2; honest typesetting note (double pdflatex; proof-assistant / numerical / referee not asserted); SOURCE_DATE_EPOCH=1792627200. held

PC20-IC-3 New Section : Imperfect Records, Approximate Reset, and Heralded Access; guiding principle printed. held

PC20-IC-4 Four conditional theorems: thm:imperfect-flag, thm:approximate-reset-spectrum, thm:private-key-resource, thm:heralded-cut; optional cor:coarse-flag; each followed by textbackslash Boundary. held

PC20-IC-5 Four research conjectures (not theorems): conj:record-refinement, conj:spectral-reset-realization, conj:reservoir-access, conj:recovery-reset-frontier; four prior conjectures retained; manual StandingBoundary after each new conjecture. held

PC20-IC-6 Distinctions printed: imperfect-flag ≠ horizon model; approximate-reset ≠ free under energy constraints; private-key heat = key-reset not encryption cost; heralded-cut exact only; shredder floor ≠ Gibbons–Hawking. held

PC20-IC-7 Update thm:release, debug ledger, stable labels, and unexecuted manifest; freeze v1 SHA in app:sha. Do not re-expand thm:reach/thm:accel-reach/Big01 epoch ledger; do not import gaussian-cross/gaussian-cov as cosmological premises. held

PC20-IC-8 Abstract 150–200 words; declaredPath=null; wired:false; nothing registered or pushed. held

DC20-IC-1 StandingBoundary retained after every new theorem/corollary; manual reminder after each new conjecture. held

DC20-IC-2 New labels present: sec:imperfect-access, thm:imperfect-flag, cor:coarse-flag, thm:approximate-reset-spectrum, thm:private-key-resource, thm:heralded-cut, and the four new conjectures. held

DC20-IC-3 Theorems remain conditional deductions; conjectures remain research claims; no proof transfer from cited parents; no conjecture promoted. held

DC20-IC-4 Opens held: O.1 NOT_CLOSED; Gate0/ST4b NOT_SUPPLIED; Dom(T) undetermined; J2 unresolved; XVI-N not discharged; XV-B not opened; Problems 6/7/9 named, not solved. held

DC20-IC-5 Abstract mentions new imperfect-access theme, four theorems, four new research conjectures; prior floors and four prior conjectures retained; non-claims held. held

DC20-IC-6 Debug ledger rows for coarse flags, spectral reset, key entropy, heralded exact return. held

DC20-IC-7 PC20-IC / DC20-IC held; request t392u. held

DC20-IC-8 declaredPath=null; wired:false; registration unexecuted. held

bottomrule

Plan corrections PC21-IC and draft verdicts DC21-IC

toprule ID Item Status

midrule endhead PC21-IC-1 Retain StandingBoundary / textbackslash Boundary after every theorem and corollary; manual StandingBoundary after the conjecture table and after the short-scope block; do not close O.1 / Gate0 / ST4b / Dom(*T*) / J2 / Problems 6/7/9; XV-B not opened; XVI-N not discharged; Host-A stays NAMED_COMPARISON_ONLY with {GI, CB}. held

PC21-IC-2 Author unchanged; date 28 October 2026; request t393u; Version 3; freeze v2 SHA; SOURCE_DATE_EPOCH=1792627200; retain PC20-IC/DC20-IC and append this table. held

PC21-IC-3 Keep every local theorem (floors, imperfect-access quartet, thermal underdetermination, rank obstruction, flagged Weyl, horizon/cosmic labels). Nothing promoted from conjecture to theorem. held

PC21-IC-4 Consolidate sibling material: shorten thm:reach/thm:accel-reach/thm:nonaccel (and optionally horizon/Big01 comparison proofs); keep thm:epoch-counterexample in full; cite I-IB/II2VI/VII/Big01 as style only. held

PC21-IC-5 Add tab:parent-comparison after the routing table; tighten thm:imperfect-flag and thm:approximate-reset-spectrum proofs; add private-key / thermal-underdetermination heat remark (no universal heat-per-qubit law). held

PC21-IC-6 Compact eight conjectures into tab:conjectures plus short scopes; clock appendix one-line comparison to lem:no-ccr / thm:tradeoffdiamond; debug-ledger row "A parent label is a local premise." held

PC21-IC-7 Update thm:release, unexecuted manifest, and app:sha; declaredPath=null; wired:false; nothing registered. held

PC21-IC-8 Abstract 150–200 words with consolidation theme (parent table, shortened cosmic, compact conjectures); opens held. held

DC21-IC-1 PC20-IC/DC20-IC retained; PC21-IC/DC21-IC held; request t393u. held

DC21-IC-2 New/changed labels: tab:parent-comparison, tab:conjectures, app:pc21; eight conjecture labels retained as conjectures. held

DC21-IC-3 Cosmic block shortened with labels retained; thm:epoch-counterexample full; imperfect proofs tightened; parent table present. held

DC21-IC-4 Opens held; no Page/island/bounce/vacuum-energy selection; shredder floor ≠ Gibbons–Hawking; no conjecture promoted. held

DC21-IC-5 declaredPath=null; wired:false; registration unexecuted; v1 and v2 SHAs frozen. held

bottomrule

SHA freeze and build record

sheet: Series-IC-Enter-A?version: 4?request: t393u?date: 2026-10-28?author: Ngo Cheung?SOURCE_DATE_EPOCH: 1792627200?FORCE_SOURCE_DATE: 1?wired: false?declaredPath: null?PC: PC20-IC + PC21-IC?DC: DC20-IC + DC21-IC?frozen_v1_tex_sha256:? 37bc0e67a15038730802366a73529f9a294677c1031f4f16e6eb00224f880c61?frozen_v2_tex_sha256:? 9b3b98c337f9fab1a3e4826c6d2f81f4a9fcffc00e500fa44a93f7ef6eb8e293

Successful double pdflatex under the recorded epoch **is** asserted for this deliverable. Proof-assistant checks, independent numerical execution, referee review, and application registration are **not** asserted. Nothing is registered or pushed.

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